LIQUID DISTRIBUTION IN REACTORS WITH RANDOMLY PACKED POROUS BEDS*

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Liquid distribution is studied in a randomly packed porous bed. The proposed mathematical model of liquid distribution has been proved to be satisfactory by comparison of theoretical distribution curves with experimental data.

Liquid distribution in the random packing of columns or reactors with wetted beds has a considerable effect on heat or mass transfer *i.e.* also on the over-all efficiency of these devices[1-6]. Majority of the published papers considering the liquid distribution in a bed is oriented to non-porous packings. In this paper is studied the liquid distribution in a randomly packed bed of porous catalyst. This contribution is related to studies published earlier[1,2] in which descriptions of the apparatus and the method used for measurement of the spreading coefficient have been given.

THEORETICAL

It has been proved experimentally[6] that the liquid flow through a randomly packed porous bed at its wetting by a central source can be divided into three regions.

1) Region of an infinite bed of random packing. This region exists at low bed heights when the liquid is still not reaching the wall of the reactor. The wetting density (*i.e.* the liquid volume passing through a unit area in a unit of time) in dependence on radial distance in the reactor is fitting approximately the Gauss's probability curve (Fig. 1, curve A). The mathematical description of liquid distribution for this flow situation is relatively simple[2]. The spreading coefficients D for studied packings with porous and non-porous character in the region of an infinite bed of packing are given in the preceding paper[2].

2) Region of finite bed of random packing (transition). This region can be approximately expected to exist[6] from the height of catalytic packing $z = 0{\cdot}042 d_k^2/d_p$. Liquid distribution in this case is already affected by the reactor wall (Fig. 1, curve B) and its description is rather complex.

3) Region of uniform liquid distribution. On basis of a number of measurements up to bed heights 5 m it has been proved that distribution of wetting densities in this region (for $d_k/d_p >$ > 25 at $T \approx 0{\cdot}37$) is about approximately uniform in the whole radial cross-section of the reactor

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and is independent of the type of liquid source employed as well as of the increasing height of packing. The experimentally determined dimensionless wetting density F is constant and equals about to 1·0 (Fig. 1, curve C). Also the wall flow *i.e.* the part of liquid flowing downward the reactor wall in this region of equilibrium distribution has steadied on a final value independent of the type of the source liquid as well as of the increasing height of packing.

On basis of relations given in literature for mathematical description of liquid distribution in separation processes[3] taking place on non-porous packings as the most suitable model was chosen that by Cihla and Schmidt[7]. These authors have proved that this problem can be, under certain assumptions, transformed to the solution of partial differential equation with a suitable boundary condition characterizing the effect of walls of the equipment on distribution. The given equation has for the case of axially symmetrical cylindrical system the form

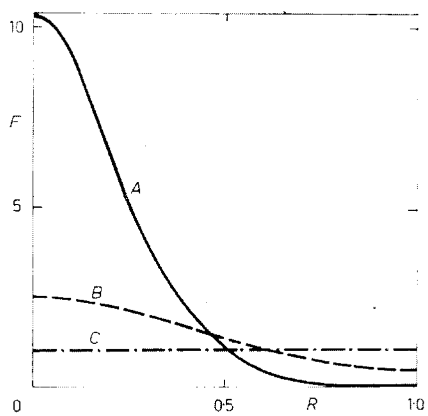

FIG. 1

Liquid Distribution in Dependence on the Bed Height at Wetting by a Point Central Source

A 0·2 m; B 0·8 m; C 4·0 m.

TABLE I

Boundary Conditions (*2a*) Characterizing Various Types of Liquid Source Applied

Liquid source	Form of boundary condition		
Point central hypothetical	$f(r, 0) = 0$ $\lim_{z \to 0} f(0, z) = +\infty$	$0 < r \leq a$	(*3*)
Circular shower	$f(r, 0) = 0$ $\lim_{z \to 0} f(r_1, z) = +\infty$	$0 \leq r \leq a \; r \neq r_1$	(*4*)
Uniform	$f(r, 0) = f_0$	$0 \leq r \leq a$	(*5*)
Wall	$f(r, 0) = 0$ $W(z) = W_0$	$z = 0$	(*6*)

$$\frac{\partial f(r, z)}{\partial z} = D\left[\frac{\partial^2 f(r, z)}{\partial r^2} + \frac{1}{r}\frac{\partial f(r, z)}{\partial r}\right]. \qquad (1)$$

The boundary conditions necessary for solution of Eq. (*1*) can be schematically described by the relations

$$f(r, z) = F_1(r) \quad \text{for} \quad z = 0\,; \qquad (2a)$$

$$\frac{\partial f(r, z)}{\partial r} = 0 \quad \text{for} \quad r = 0\,; \qquad (2b)$$

$$\frac{\partial f(r, z)}{\partial r} = F_2(z) \quad \text{for} \quad r = a\,. \qquad (2c)$$

The boundary condition (*2a*) is given by the distributor employed. Its shape is for the basic types of distributors given in Table I. Condition (*2b*) is representing the validity of axial symmetry. The third condition (*2c*) is characterizing the effect of the equipment walls in the region of finite bed of packing on the liquid distribution (in the region of infinite bed of packing $f(a, z) = 0$ holds).

Individual mathematical models based on partial differential equation (*1*) differ first of all in the definition of condition (*2c*) — Table II. In this study we are studying how suitable it is to apply Eq. (*1*) and the boundary condition (*2c*) for mathematical description of liquid distribution in trickle bed reactors.

As the boundary condition, according to Dutkai and Ruckenstein[8-10], at the solution of Eq. (*1*) becomes identical[6] with the condition according to Kolář and Staněk[9] only the remaining three boundary conditions have been verified. The final forms of solution of the diffusion equation (*1*) for various types of distributors and boundary conditions of the quoted authors are not

TABLE II

Boundary Condition (*2c*) Characterizing the Effect of Wall on Liquid Distribution in the Region of Limited Packed Bed

Author	Form of boundary condition	
Cihla–Schmidt[7]	$\operatorname{grad} f(a, z) = 0$	(*7*)
Porter–Jones[8,12]	$f(a, z) = k_2 W(z)$	(*8*)
Kolář–Staněk[9,11]	$-2\pi a D \dfrac{\partial f(a, z)}{\partial r}\,\mathrm{d}z = 2\pi a\beta[f(a, z) - \gamma\ W(z)]\,\mathrm{d}z$	(*9*)
Dutkai–Ruckenstein[10]	$-2\pi D \left(\dfrac{\partial f(r, z)}{\partial r}\right)_{r=a-\delta} = k_3 f(r, z)_{r=a-\delta} - k_3' W(z)$ $-2\pi D \left(\dfrac{\partial f(r, z)}{\partial r}\right)_{r=a-\delta} = \dfrac{\partial W(z)}{\partial z}$	(*10*)

due to a considerable complexity, derived here but their derivations are given in papers[6,7,11,12]. For simplicity are in Table III given solutions of Eq. (*1*) with the boundary conditions according to Cihla and Schmidt[7], Kolář and Staněk and Porter and Jones[8] for basic types of distributors. In these solutions are included several parameters which are determined as follows:

Determination of the dimensionless number T. The dimensionless number T is given by the relation

$$\mathrm{T} = Dz/a^2 \,. \tag{21}$$

For a constant radius of the reactor and the spreading coefficient D for the given

TABLE III

Solution of Eq. (*1*) for Basical Types of Distributors by Use of the Boundary Condition (*2c*) of the Given Authors

Liquid source	Forms of solution with the boundary condition (*2c*) used	
	Cihla–Schmidt[7] [a]	
Point central hypothetical	$F = 1 + \sum_{n=1}^{\infty} \frac{\mathrm{J}_0(q_n R)}{\mathrm{J}_0^2(q_n)} \exp(-q_n^2 \mathrm{T})$	(*11*)
Circular shower	$F = 1 + \sum_{n=1}^{\infty} \frac{\mathrm{J}_0(q_n R_1)\,\mathrm{J}_0(q_n R)}{\mathrm{J}_0^2(q_n)} \exp(-q_n^2 \mathrm{T})$	(*12*)
Uniform	$F = 1 + 2\sum_{n=1}^{\infty} \frac{\mathrm{J}_1(q_n)\,\mathrm{J}_0(q_n R)}{q_n\,\mathrm{J}_0^2(q_n)} \exp(-q_n^2 \mathrm{T})$	(*13*)
Wall[d]	$F = 1 + \sum_{n=1}^{\infty} \frac{\mathrm{J}_0(q_n R)}{\mathrm{J}_0(q_n)} \exp(-q_n^2 \mathrm{T})$	(*14*)
	Porter–Jones[8] [b]	
Point central hypothetical	—	
Circular shower	—	
Uniform	$F = \frac{\mathrm{K}}{1 + \mathrm{K}} - 4\mathrm{K} \sum_{n=1}^{\infty} \frac{\mathrm{J}_0(q_n R)}{(q_n^2 + 4\mathrm{K} + 4\mathrm{K}^2)\,\mathrm{J}_0(q_n)} \exp(-q_n^2 \mathrm{T})$	(*19*)
Wall[a]	$F = \frac{\mathrm{K}}{1 + \mathrm{K}} + 4\mathrm{K}^2 \sum_{n=1}^{\infty} \frac{\mathrm{J}_0(q_n R)}{(q_n^2 + 4\mathrm{K} + 4\mathrm{K}^2)\,\mathrm{J}_0(q_n)} \exp(-q_n^2 \mathrm{T})$	(*20*)

TABLE III
(*Continued*)

	Kolář–Staněk[9] [c]	
Point central hypothetical	$F = \frac{C}{1+C} \sum_{n=1}^{\infty} \frac{[(q_n^2/B) - 2C]^2 J_0(q_n R)}{\{[(q_n^2/B) - 2C]^2 + q_n^2 + 4C\} J_0^2(q_n)} \exp(-q_n^2 T)$	(*15*)
Circular shower	$F = \frac{C}{1+C} \sum_{n=1}^{\infty} \frac{[(q_n^2/B - 2C]^2 J_0(q_n R) J_0(q_n R_1)}{\{[(q_n^2/B) - 2C]^2 + q_n^2 + 4C\} J_0^2(q_n)} \exp(-q_n^2 T)$	(*16*)
Uniform	$F = \frac{C}{1+C} \sum_{n=1}^{\infty} \frac{2[(q_n^2/B - 2C] J_0(q_n R)}{\{[(q_n^2/B) - 2C]^2 + 4C\} J_0(q_n)} \exp(-q_n^2 T)$	(*17*)
Wall[d]	$F = \frac{C}{1+C} - C \sum_{n=1}^{\infty} \frac{2[(q_n^2/B) - 2C] J_0(q_n R)}{\{[(q_n^2/B) - 2C]^2 + q_n^2 + 4C\} J_0(q_n)} \exp(-q_n^2 T)$	(*18*)

[a] q_n are roots of Eq. $J_1(q_n) = 0$. [b] q_n are roots of Eq. $q_n J_0(q_n) + 2KJ_1(q_n) = 0$. [c] q_n are roots of Eq. $[(2C/q_n) - (q_n/B)] J_1(q_n) + J_0(q_n) = 0$. [d] As the wall is, according to the mathematical model by Cihla and Schmidt, a total reflector (the wall flow does not originate) for the wall source the solution was in this case substituted by the solution for circular shower with $R_1 = 1$.

experimental arrangement, the value of the number T is dependent on the variable height of the bed z.

Determination of the dimensionless number C. The necessary condition for solution of the diffusion equation (*1*) with an arbitrary boundary condition characterizing the effect of walls is that the parameter of the boundary condition is not a function of the bed height. Evaluation of the number C is based on this assumption. By taking the limit $z \to \infty$ in the relation for calculation of liquid distribution at wetting the bed by a uniform (*17*) or some other liquid source, the relation is obtained which includes only the looked for parameter C and which gives the part of total liquid flow rate flowing downward the packing at its "infinite" height

$$f(r, \infty)/f_0 = C/(1 + C) . \tag{22}$$

The equilibrium wall flow is given on basis of the material balance by the dependence

$$W(\infty)/\pi a^2 f_0 = 1/(1 + C) . \tag{23}$$

By arranging Eq. (*23*), the relation for calculation of the dimensionless number C from the equilibrium wall flow rate is obtained and is expressed in % of the over-all liquid flow rate

$$C = [100/W_p(\infty)] - 1\,. \tag{24}$$

When the actual value of number C is determined experimentally the "infinite" bed height is substituted by such finite height from which the wall flow becomes constant. The error in determination of this number at wetting beds with lower heights of packing than is the limiting height (corresponding to the beginning of the equilibrium liquid distribution) is reaching considerably higher values at wetting by other types of distributors than is the uniform one.

Determination of the dimensionless number B. The number B is evaluated by the least square method. At a constant calculated value of number C and a variable value of number B are for various packing heights calculated such distribution curves which have the smallest sum of second powers of deviations from the experimentally found points. But it has resulted that for higher values of parameter C (>10), *i.e.* also in this case, the model by Kolář and Staněk is in a wide range little parametrically sensitive to the value of number B which represents the system dynamics (Fig. 2).

Determination of the dimensionless number K. This number is in the limit for $z \to \infty$ identical with the parameter C according to Kolář and Staněk. Its value, which is identical with the value of parameter C, is thus determined according to the already described procedure.

EXPERIMENTAL

Apparatus and measurements. The experimental apparatus and its description has been already given in the preceding paper[1]. Beside the central point source and circular shower another two types of liquid source were used — the uniform and wall types.

By use of the wall distributor it was possible to introduce the whole liquid feed on the column wall through holes on the side of the source. Their number (63—205) and their diameter (1·0 mm) was given by the initial wetting density (1—15 m^3/m^2 h).

The uniform liquid source which had good spreading properties was formed by a disc having the same diameter as the column on the bottom of which were in angles of a hexagonall mesh situated 121 holes with ID 0·3 or 0·5 mm. The maximum initial wetting density was for the source 11 m^3/m^2 h.

In the measurement was used the same basical procedure as in studies[1,2]. The liquid feed was water, packing Nickel on Kieselguhr[2]. All the experiments were performed in the region of diffusion mechanism of liquid transfer which for the given conditions was proved[6] to exist.

Evaluation of experimental data. In total, more than 30 thousands of experimental data presented in papers[6,14–17] were evaluated. Even though the liquid distribution in the random packing is a process of a statistical character very good reproducibility of results has been reached for porous packings. The variance of experimental data for a newly packed bed which is characterized by the standard deviation is for various types of distributors and variable heights of the porous packing in the range from 15 to 25%. The variance has decreased with increasing height of the packing and with better uniformity of liquid flow (uniform liquid source)[6]. With regard to a great increase in the number of experiments which would be necessary for obtaining a greater accuracy of these types of measurements (in minimum five repeated measurements of which the average is taken) the experiments were not repeated. Therefore it is necessary to realize, especially in comparing the experimental data with the theoretical curves, the size of the error which is affecting the measurements.

The given ratios f/f_0 are representing the average of the experimentally determined dimensionless wetting densities on the given radius for the region of f_0 from 1 to 15 m^3/m^2 h (with the uniform source for the region from 1 to 11 m^3/m^2 h).

RESULTS AND DISCUSSION

Individual mathematical models proposed for distribution of liquid on a random porous packing were verified by comparison of theoretical distribution curves obtained by evaluation of Eq. (*1*) with the experimental data. For solution of Eq. (*1*) at wetting by a point central source, the boundary condition of hypothetical source was used as its suitability for the given experimental arrangements had been proved (the area of the source is negligible in comparison with the cross-sectional area of the reactor). The calculations were performed on the computer Tesla 200.

Values of the basical numbers which were obtained by methods described earlier on basis of a considerable number of experimental data are for the given arrangement as follows: $D = 0{\cdot}001939$ m; $B = 1{\cdot}5$; $C = 13{\cdot}3$.

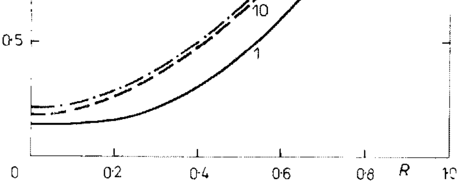

FIG. 2

Distribution Curves of Wall Source Calculated from Eq. (*18*) for Various Values of Number B and for C = 13·3 and $z = 0{\cdot}4$ m

——— B = 1, – – – B = 10, –·–·– B = = 100.

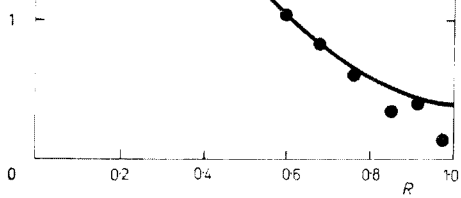

FIG. 3

Comparison of Distribution Curve for the Central Point Source Calculated from Eq. (*15*) with the Experimental Points (B = 1·5, C = 13·3, $z = 0{\cdot}8$ m)

TABLE IV

Variance of Experimental Data from Theoretical Curves of Studied Mathematical Models at Wetting by 4 Types of Liquid Sources in the Region of Limited Bed Height and Equilibrium Liquid Distribution

Source: I Point central, II circular shower, III uniform, IV wall.

Liquid source	Packing height m	Mean standard deviation		
		Cihla–Schmidt	Kolář–Staněk	Porter–Jones
I	0·4—5·0	0·2761	0·2691	—
II	0·4—1·2	0·3115	0·3082	—
III	0·3—5·0	0·1446	0·1286	0·1502
IV	0·4—5·0	0·2415	0·1652	1·0232

The mathematical models were verified under wetting the catalytic packing by four types of liquid source (point central, circular shower, uniform and wall) in the region of limited bed of random packing (0·4 to 1·8 m high) as well as of the uniform liquid distribution (3·0 to 5·0 m). The deviation of experimental points from the theoretical curves was indicated according to the value of standard deviations (Table IV) which had been calculated from the relation

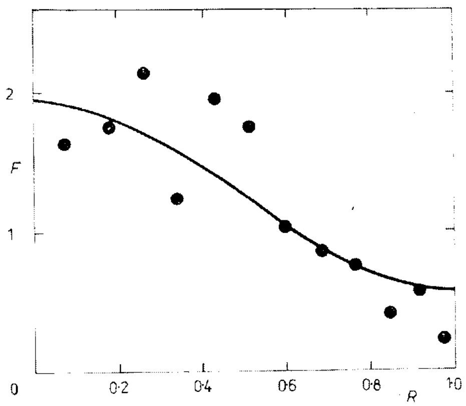

FIG. 4

Comparison of Distribution Curve of Circular Shower Source Calculated from Eq. (*16*) with the Experimental Points (B = 1·5, C = 13·3, z = 0·8 m)

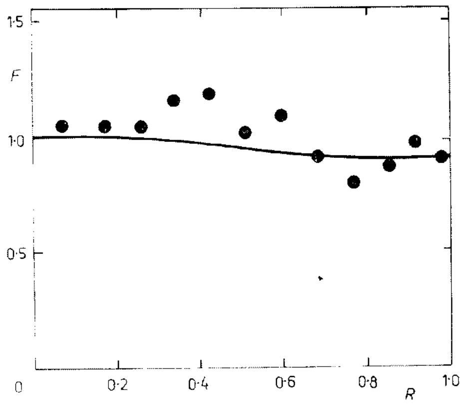

FIG. 5

Comparison of Distribution Curve of Uniform Source Calculated from Eq. (*17*) with the Experimental Points (B = 1·5, C = 13·3, z = 0·9 m)

$$\sigma = \pm \left(\sum_{i=1}^{n} (F_{t,i} - F_{e,i})^2/n\right)^{1/2} . \quad (25)$$

On the basis of results given in Table IV is obvious that the two-parameter condition of Kolář and Staněk[9] is best expressing the effect of the wall on distribution at the studied boundary conditions. This condition makes possible to obtain a sufficiently accurate description of liquid distribution in the randomly packed porous bed by solving the partial differential Eq. (*1*) which is also confirmed by dependences plotted in Figs 3 to 7.

The boundary condition of Cihla and Schmidt[7], though it is not fully in agreement with the real behaviour of the reactor wall (with increasing bed height the wall flow forms and the wall does not behave as a total reflector), can be also classified as suitable for quantitative evaluation of liquid distribution in a porous packing under the assumption of lower equilibrium wall flow (less than 10%). In comparison of the model by Kolář–Staněk with that by Cihla and Schmidt on basis of the Snedercer's[18] test only for one out of 44 data sets it can be concluded (wall source, bed height 0·8 m) that the difference between the tested variances is at the 95% probability statistically significant. Slightly greater variance of experimental data from theoretical curves for the Cihla–Schmidt model can be in some cases outweighted by simplicity of solution of this model, which beside the spreading coefficient D does not include

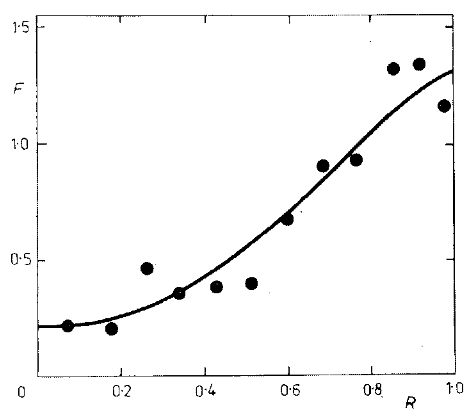

FIG. 6

Comparison of Distribution Curve of Wall Source Calculated from Eq. (*18*) with the Experimental Points (B = 1·5, C = 13·3, z = 0·8 m)

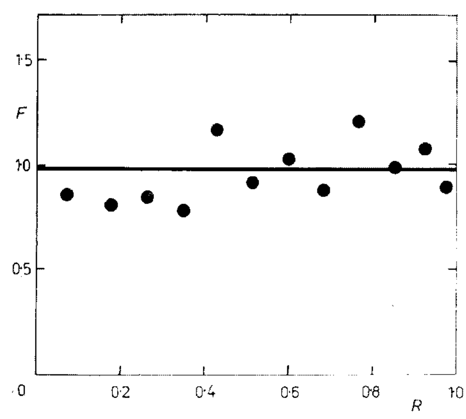

FIG. 7

Comparison of Distribution Curve of Point Central Source Calculated from Eq. (*15*) with the Experimental Points (B = 1·5, C = 13·3, z = 4·0 m) any other constant. The boundary condition according to Cihla and Schmidt will be the more satisfactory, the lower will be the wall flow in the equilibrium state.

The least suitable has proved to be the boundary condition by Porter and Jones[8]. First of all, at wetting the wall by the liquid source the assumption of equilibrium between the wetting density at the wall and the wall flow is unsatisfactory. It can be expected that this boundary condition will enable description of the liquid distribution with a greater accuracy in systems where the steady wall flow will soon become significant (in systems with small value of the ratio d_k/d_p).

LIST OF SYMBOLS

a	radius of the reactor or column (m)
B	number characterizing transfer of liquid into the wall
C	spreading number
D	spreading coefficient (m)
d_k	diameter of column or reactor (m)
d_p	diameter of the packing element (m)
$F = f/f_0$	dimensionless wetting density
$F_1(r)$	function determining the initial liquid distribution
$F_2(z)$	function determining the liquid distribution at the reactor wall
f	wetting density ($m^3 m^{-2} h^{-1}$)
f_0	initial wetting density ($m^3 m^{-2} h^{-1}$)
f_∞	wetting density in the region of equilibrium distribution ($m^3 m^{-2} h^{-1}$)
J_0	Bessel's function of 1st kind, zero order
J_1	Bessel's function of 1st kind, first order
k_2, k_3, k'_3	distribution constants (m, m^{-1})
q_n	root of transcendent equations
$R = r/a$	dimensionless radius
r	radius in cylindrical coordinates (m)
$T = Dz/a^2$	dimensionless form of the spreading coefficient
W	wall flow ($m^3 h^{-1}$)
$W_0 = f_0 \pi a^2$	initial wall flow ($m^3 h^{-1}$)
W_p	relative wall flow (related to the total flow rate) (%)
$W_p(\infty)$	relative wall flow in the region of equilibrium distribution (%)
z	bed height (m)
β	coefficient of liquid transfer into the wall
γ	spreading coefficient (m^{-2})
δ	periphery of packing where the diffusion mechanism of liquid transfer already does not hold (m)
φ	angular coordinate in cylindrical coordinates (radian)

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